# Equivalent annual annuity vs. replacement chain approach for mutually exclusive investment projects

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## ABSTRACT

This teaching note will confirm that the two standard textbook approaches to evaluate mutually exclusive investment projects, equivalent annual annuity and replacement chain, will both work correctly if the mutually exclusive projects have identical discount rates, i.e., they are projects of similar risk characteristics. Textbook authors typically recommend the use of the equivalent annual annuity approach. However, the note will also show, via numerical illustrations, that the classic textbook methods may recommend incompatible decisions between the two approaches if the projects have different discount rates because their risk characteristics are different. In such a case, it will be argued that the replacement chain approach is superior because it directly measures the value enhancement associated with a project.

**INTRODUCTION**

The capital budgeting decision is one of the major business issues that the chief financial officer of a corporation must address. Of the menu of possible investment projects facing the firm, which are to be accepted and which are to be rejected? Finance practitioners have developed analytical methodologies to assist with the accept/reject decisions for projects with a wide range of characteristics. The purpose of this teaching note is to consider a particular configuration of available projects: mutually exclusive projects with different lives. Mutually exclusive projects are projects for which it makes sense to accept only one of the projects. For example, if a florist is considering either the purchase of a truck or a van to deliver flowers, the florist will not need the truck if the van is purchased and vice-versa. That is, in this case, it makes sense to accept only one of the projects. Further, projects have different lives if the durations of the projects are different. Given that the projects are mutually exclusive and have unequal lives, the analyst can no longer rely solely on the Net Present Value Method (NPV), but must consider replicating the projects to a common end date. For example, if project #1 has a 3 year time horizon and project #2 has a 5 year time horizon, it will be necessary to analyze the investment opportunities over a 15 year period.

Two modes of analysis have been developed to capture this replication analytically: (1) Effective Annual Annuity (EAA) and (2) Replacement Chain (RC). These procedures are well-known and presented in virtually every financial management textbook:

EFFECTIVE ANNUAL ANNUITY (EAA)

- Compute NPV of project – do not consider replication
- Convert the NPV to an annuity with time horizon equal to the time horizon of project - EAA
- Choose mutually exclusive project with highest EAA

REPLACEMENT CHAIN (RC)

▸ Replicate the projects to a common end date
▸ Find the present vale (PV) of each of the expanded NPV streams
▸ Choose the mutually exclusive project with the highest PV

After describing the procedures and presenting numerical examples, textbook authors typically assert that the procedures will make the same accept/reject decisions. As indicated in Table 1 (Appendix), the relevant language used in three prominent Financial Management texts is provided.

The textbook approaches will work correctly if the mutually exclusive projects have identical discount rates, i.e., they are projects of similar risk characteristics. In such cases, EAA and Replacement Chain are consistent with each other and either approach may be used. However, this teaching note will also show, via a numerical illustration, that the two methods may recommend incompatible decisions if the projects have different discount rates because their risk characteristics are different. In such a case, most financial economists would argue that the

replacement chain approach is superior because it directly measures the value enhancement associated with a project.

Consider the two mutually exclusive projects presented in Table 2 (Appendix). NPV, EAA and Replacement Chain were calculated for both projects using the standard textbook formulae. Table 2 (Appendix) presents the standard textbook presentation. Note the key characteristics of the projects:

- Different durations (3 yrs. vs. 5 yrs.)
- Identical discount rates (3% for both projects)
- NPV is not appropriate (Project #2 has higher NPV)
  - Project #1 NPV = $106.43; Project #2 NPV = $179.44
- EAA and Replacement Chain consistent - recommend Project #2
  - Project #1 EAA = $37.63; Project #2 EAA = $39.18 => select Project #2
  - Project #1 RC = $449.18; Project #2 RC = $467.76 => select Project #2

Both EAA and RC "correctly" choose Project #2. While the standard NPV approach also correctly chooses Project #2, this result is not reliable. The reader may consult any of the textbook references provided herein for examples in which the standard NPV calculation recommends a different project than either the EAA or RC. The weakness with the standard NPV approach is that completely ignores the complications associated with the different project durations.

Now consider the same two mutually exclusive projects, but assume that Project #2 is riskier than Project #1 and therefore requires a higher discount rate, see Table 3 (Appendix). Table 3 (Appendix) is more complex than the standard textbook presentation because the discount rates (and projects' riskiness) are different. Note the key characteristics of the projects:

- Different durations (3 yrs. vs. 5 yrs.)
- Different discount rates
  - Project #1 discount rate = 3%; Project #2 discount rate = 6%
- NPV is not appropriate (Project #2 has higher NPV)
  - Project #1 NPV = $106.43; Project #2 NPV = $161.04
- EAA and Replacement Chain **inconsistent**
  - Project #1 EAA = $37.63; Project #2 EAA = $38.23 => select Project #2
  - Project #1 RC = $449.18; Project #2 RC = $371.30 => select Project #1

EAA and RC provide inconsistent recommendations; under these circumstances, select Project #1 based on RC. Also note that in this case the standard NPV makes the incorrect choice, recommending project #2.

Table 3 (Appendix) provides a counter example to the proposition that EAA and RC provide consistent results. As such, the analyst must choose either EAA or RC. Financial economists will prefer RC because this approach measures the value of the projects directly.

**SUMMARY**

It is quite common for a firm to face investment opportunities containing mutually exclusive projects with unequal lives. If the projects have different risk characteristics, and therefore different discount rates, it is possible that the EAA and replacement chain approaches provide different recommendations. Nevertheless, the leading textbooks assert that either procedure may be used or, alternatively, that the EAA should be used. This teaching note has emphasized that, if project risk characteristics are different, the procedures may well provide

inconsistent accept/reject recommendations.  In such cases, this teaching note suggests that the replacement chain approach is best suited because it directly measures value added by the projects.

It is important to emphasize that the conclusions herein apply only to the very special case in which projects are mutually exclusive, have unequal lives (and therefore are to be replicated across time), and have different risk characteristics.  In such a case, students must be aware that the standard textbook  solution methodology may lead to non-optimal decision making.

**REFERENCES**

Brealey, R., Myers, S. & A. Marcus (2012) "Fundamental of Corporate Finance", McGraw-Hill, 7th edition.

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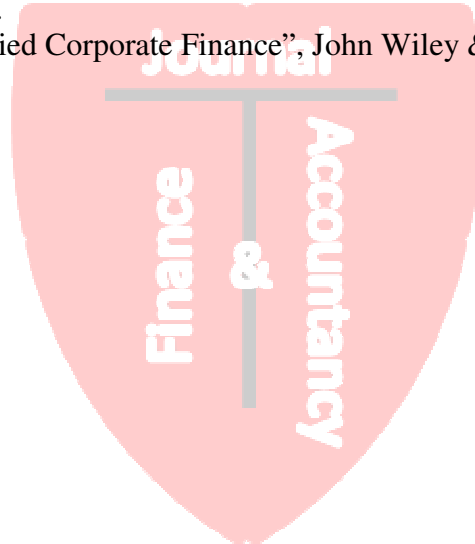
**APPENDIX**

TABLE 1

| Brigham & Ehrhardt (2004, 2009) | "two different approaches can be used to correctly compare projects" |
|---|---|
| Damodaran (2011) | "it *(EAA)* will always lead to the same decision rules as the replacement method" |
| Brealey, Myers & Marcus (2012) | Present only EAA as the appropriate solution methodology |

TABLE 2

| | PROJECT #1 | PROJECT #2 |
|---|---|---|
| | | |
| Project Duration | 3 yrs. | 5 yrs. |
| Annual Cash Flows | $50.00 | $50.10 |
| Discount Rate | **3%** | **3%** |
| Initial Investment | $35.00 | $50.00 |
| | | |
| NPV | $106.43 | $179.44 |
| | | |
| EAA | $37.63 | **$39.18** |
| Replacement Chain (RC) | $449.18 | **$467.76** |

Table 3

| | PROJECT #1 | PROJECT #2 |
|---|---|---|
| | | |
| Project Duration | 3 yrs. | 5 yrs. |
| Annual Cash Flows | $50.00 | $50.10 |
| Discount Rate | **3%** | **6%** |
| Initial Investment | $35.00 | $50.00 |
| | | |
| NPV | $106.43 | $161.04 |
| | | |
| EAA | $37.63 | **$38.23** |
| Replacement Chain (RC) | **$449.18** | $371.30 |