A regime-switching model of exchange rate volatility in Mexico

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ABSTRACT

The Switching ARCH (SWARCH) model of Hamilton and Susmel (1994) is applied to the weekly percentage change of the Mexican Peso-U.S. Dollar exchange rate. The existence of two statistically significant volatility states is documented. Five episodes of high volatility in the Peso were found, the first one linked to a domestic financial crisis (the Peso crisis of 1994-1995) and three more associated to international financial crises (the Russian and Brazilian crises of 1998 and 1999, the global financial crisis of 2008-2009 and the European debt crisis of 2011-2013). The last episode of high volatility occurred in June 2013 and was linked to the prospect of imminent tapering of the Federal Reserve's policy of quantitative easing (QE) program. The low volatility state has an expected duration of 109 weeks whereas the high volatility state is shorter and has an expected duration of 41 weeks.

Keywords: Switching ARCH model, SWARCH model, Exchange rate volatility, Mexican Peso

INTRODUCTION

Since the introduction of the Autoregressive Conditionally Heteroskedastic (ARCH) model by Engle (1982), the ARCH models have been successfully employed in the analysis of financial time series and have been widely used to model the volatility in financial markets. A typical result of the ARCH models is that they imply a very high degree of persistence in the conditional volatility. This outcome may be artificial if structural changes in the conditional variance exist. Uncommon events such as financial crises, recessions or changes in government policies may cause these structural changes. Lamoreux and Lastrapes (1990) argue that the high level of persistence found in ARCH models might be due to the existence of these structural changes, which are not included in traditional ARCH models. Incorporating deterministic switches in the conditional variance results in a significant decrease in the level of persistence. Hamilton and Susmel (1994) developed a regime-switching ARCH model known as the SWARCH model which incorporates regime switches into the conditional variance caused by unusual events such as financial crises and recessions. They applied the SWARCH model to the analysis of stock returns in the United States and found that it implies a reduced level of volatility persistence than the traditional ARCH models that do not account for regime switches.

The purpose of this study is the application of the SWARCH model to the volatility of the Peso exchange rate in the sample period 1995-2013, a period that includes several currency and financial crises such as the the Peso crisis of 1994-1995, the Asian financial crisis of 1997, the Russian and Brazilian crises of 1998 and 1999, the Argentinean economic crisis of 2001-2002 and the global financial crisis of 2008. The purpose of this research is to look for statistically significant evidence of regime-switching volatility in the Peso exchange rate. The most innovative feature of the SWARCH model is that it allows the objective dating of the switches in volatility regimes. Two distinct regimes in the time series of exchange rates in Mexico were identified and dated. It was also found that the SWARCH model appropriately characterizes exchange rates in Mexico and that the inclusion of structural breaks substantially decreases the persistence of the conditional volatility. Finally, it was found that the volatility regime switches tend to coincide with local and international financial crises.

LITERATURE REVIEW

Since Hamilton (1989) introduced Markov regime-switching models, they have been widely used in the analysis of financial time series. In the particular case of the Mexican Peso exchange rate, a few papers have looked at its behavior in the context of a regime-switching framework. Tovar-Silos (2012) studied the joint behavior of Mexican exchange rates and stock market returns and found that a test for the presence of two regimes was statistically significant for both financial variables. He also found that short-lived high variance, depreciationary states in exchange rates were in sync with short-lived episodes of heightened volatility and negative stock market returns. In a regime-switching model without ARCH effects, Bazdresch and Werner (2005) found statistically significant evidence for the existence of two states: the first one characterized by currency appreciations and low variance, and the second one characterized by large depreciations and high variance. Cheung and Miu (2008) argued that concluding that statistically significant regimes exist may be false and instead the result of ARCH effects.

The SWARCH model has been widely used in the analysis of financial variables such as stock market returns and interest rates in Latin-American economies such as Mexico. Using a SWARCH model, Canarella and Pollard (2007) documented the existence of high volatility regimes in Mexico and five other Latin American stock markets. They also found that the changes in volatility regime are a direct consequence of financial crises. Edwards and Susmel (2001), using stock market data for a group of Latin American countries found that high-volatility episodes are short-lived lasting from two to twelve weeks. They also found strong evidence of volatility co-movement among Latin American countries. In another study, Edwards and Susmel (2003) used interest rate data for a group of Asian and Latin American countries to analyze the behavior of volatility through time. They found that high volatility episodes are short-lived, lasting from 2 to 7 weeks and that there is evidence of interest rate co-movements across countries. Diamandis (2008) used the SWARCH model to examine the dynamic behavior of stock market volatility for four Latin American markets during the financial liberalization period that extended from January 1988 to July 2006. She found evidence of the existence of more than one volatility regime and that there are three episodes of high volatility for all markets around the Asian and Russian financial crises.

Only a few research articles have applied the SWARCH model to the analysis of exchange rates. Fong (1998) applied the SWARCH model to the DM/£ exchange rate in the period 1987 to 1994 which includes the entrance and exit of the UK to the Exchange Rate Mechanism (ERM). He found that the ARCH effects are substantially less severe once structural breaks are included in the model and that volatility switched to a low state shortly after Britain's ERM entry and it remained in that state until the September 1992 ERM crisis. Brunetti et al. (2007) analyzed exchange rate turmoil in four Southeast Asian countries: Taiwan, Singapore, the Philippines and Malaysia with a Markov switching GARCH model. They found evidence of two regimes: an "ordinary" regime characterized by low exchange rates and low volatility and a "turbulent" regime characterized by high exchange rate movements and high volatility. They also found that real effective exchange rates, money supply relative to reserves, stock index returns and bank stock index returns contain information for identifying these regimes.

METHODOLOGY

Data Collection

Banco de Mexico (Mexican Central Bank) was the source of the time series data of the weekly exchange rate of the Peso. A total of 989 data values were collected for the sample period extending from January 1995 to December 2013.

Model Specification

Markov regime-switching models assume that the realizations of a time series are the outcome of one of two different probability distributions, with an explicit probability law governing the likelihood that each realization of the variable was obtained from a particular probability distribution.

One of the main objectives of this investigation is to test to determine whether the existence of high and low volatility states in the time series of exchange rates in Mexico is statistically significant. To attain this objective, the author applied the Hamilton and Susmel (1994) SWARCH model to the sample period 1995-2013 and the following null and alternative hypotheses were developed and tested:

H_0: There is no regime-switching in the ARCH effects of the Peso exchange rate.

H_1**:** There is regime-switching in the ARCH effects of the Peso exchange rate.

In order to test whether there is regime-switching in the ARCH effect of the time series of exchange rates, two econometric models are considered in this investigation. Model one is a conventional AR(1)-ARCH(q) specification which will be used as a benchmark for the identification of regime switches. In other words, the AR(1)-ARCH(q) model, is the model implied by the null hypothesis.

The following equations present the SWARCH model which is the model implied by the alternative hypothesis. For the purpose of this study, the variable x_t is defined as the Peso exchange rate at time t. Specifically, the following first-order autoregressive model for the time series of x_t is considered:

$$\Delta x_t = \mu + \Phi \Delta x_{t-1} + \varepsilon_t \tag{1}$$

where:

$$\Delta x_t = ln(x_t) - ln(x_{t-1}) \tag{2}$$

and ε_t is a random error term. An assumption of the ARCH models is that the variance of the dependent variable (exchange rates) is time-variant. Each ARCH model uses a different specification to describe the error process. In the SWARCH specification, the error process is modeled by the following equations:

$$e_t = u_t \sqrt{g(s_t)} \tag{3}$$

where u_t follows a conventional ARCH(q) process:

$$u_t = \sqrt{h_t} v_t, \; v_t \sim \text{i.i.d. } D(0, 1) \tag{4}$$

and h_t obeys the ARCH(q) process:

$$h_t = \propto_0 + \sum_{j=1}^{q} \propto_j u_{t-j}^2 \tag{5}$$

The model in equations 1 to 5 assumes that the conditional mean μ is not state-dependent. In Tovar-Silos (2014), the estimated model allowed switching means but ARCH effects were not included. The assumption of a non-switching mean reduces the computational work employed in the estimation process and allows the investigation to be concentrated on the process of the non-constant variance. In relation to the probability distribution D, the analysis will begin with the normal distribution. The results under the assumption of the normal distribution will be compared with those obtained with the student's t specification. This specification has proved to be highly relevant in the modeling process of financial data which tend to exhibit fat tails.

In equation 3, the ARCH process can be scaled by the constant variance factor $g(s_t)$. It represents the ratio of the volatility in a "high" state relative to a "low" state. The factor varies according to the state s_t which in any given point in time is either 0 or 1. In the analysis that follows, state 0 will be the low volatility regime and state 1 will be the high volatility regime. The factor $g(s_t)$ is normalized to be equal to 1 in state 0 so that in state 1, the factor represents the ratio of the volatility in a high regime and the volatility in a low regime. Note that if the number of states is reduced to 1, $g(s_t)$ is equal to 1 and the SWARCH model becomes the AR(1)-ARCH(q) model. In this sense, the AR (1)-ARCH(q) model is nested in the SWARCH model, and a traditional Likelihood Ratio test (LR) can be used to test the hypothesis of regime-switching in the ARCH effects.

The estimation of the SWARCH model also provides estimates of the fraction of time that the process stays in each regime (also known as ergodic probabilities) and the probabilities of transition between different regimes. The coefficients for each regime as well as these estimates are necessary so that the model can be used to make forecasts. The probabilities of transition from one state to another can be expressed in a matrix *P*:

$$P = \begin{bmatrix} P_{11} & P_{21} \\ P_{12} & P_{22} \end{bmatrix} \tag{6}$$

where P_{ij} represents the probability of switching to state j given that the current state is i. One common specification used in the literature is that these probabilities are constant. More complex specifications allow for these probabilities to depend on some set of fundamental variables. The ergodic probabilities can be easily computed from the transition probabilities in matrix *P*. For example, the fraction of time that the time series stays in state i is given by the ratio $1/(1-P_{ii})$. Given this specification, it is expected that the variable of study to switch from one regime to another according to the transition probabilities, and to remain in that state for several periods of time according to the ergodic probabilities.

THE RESULTS OF THIS STUDY

Graph 1 shows the time series of the percentage change of exchange rates in Mexico. The graph shows clear evidence of volatility clustering which has been defined as the tendency of volatility in the prices of financial assets to lead to more volatility. A well-documented result in empirical finance is that financial time series exhibit volatility clustering and the time series of the Peso seems to be no exception. An informal inspection of this graph permits the identification of several periods of heightened volatility, the most important being the one experienced around the Mexican Peso crisis that started on December 1994 and was extended throughout most of 1995. Another extended period of increased volatility occurred between October 1997 and January 1999, a period that included the Brazilian and the Asian financial crises. One more episode of increased volatility can be identified in October 2008 during the global financial crisis. As was explained before, a benefit of the SWARCH model is the objective identification and characterization of these periods.

Table 1 presents some descriptive statistics for the percentage change of exchange rates in Mexico. It can be observed from the table that the Mexican Peso has depreciated on average 0.09% per week during the studied period. Note also that volatility, as measured by the standard deviation, is high in the Mexican exchange rate market. The coefficient of variation (not shown) suggests that the standard deviation is more than 20 times the value of the mean. The null hypothesis that the distribution is normally distributed is strongly rejected as indicated by the Jarque-Bera test, a typical result of financial data. The extremely high kurtosis (an indication of heavy tails) is also commonly observed in this type of data. The results of the Ljung-Box test suggest that the null hypothesis of white noise residuals can be rejected at the 10% level of significance and that significant autocorrelation is present. Also, the Ljung-Box test on the squared residuals suggest that there is strong evidence against the null of no autocorrelation and provides evidence for the existence of ARCH effects in the conditional variance.

The results of the estimation of the SWARCH with 2 states and one autoregressive lag SWARCH(2,1) are presented on Tables 2 and 3. Bazdresch and Werner (2005) and Tovar-Silos (2012) have documented the presence of two states on the time series of the Peso. Also, in this research, different number of lags in the ARCH process were tried and either there was no significant increase in the log likelihood function or the model could not be estimated. For this reason, the specification used in this investigation uses only one lag. The normal and the student's t were both used as the underlying probability distributions of the error term in the estimation of the SWARCH(2,1). The estimates of the model that assumes the normal distribution specification are shown on Table 2 whereas Table 3 shows the results when the t distribution was used. The estimate of the first-order autoregressive coefficient (Φ) is negative and small in both the normal and the t distribution specifications, a result that coincides with previous studies on Mexican exchange rates such as Bazdrech and Werner (2005) and Tovar-Silos (2012). These small and negative coefficients suggest that exchange rates in Mexico are negatively correlated and that they are not easy to predict. The ARCH parameter (α_1) is relatively large and statistically significant for both distributions, taking values of .30 and .20. When comparing the log likelihood of the two specifications, it was found that the t specification is better than the normal specification in terms of the magnitude of the log likelihood function.

The estimate of the parameter g(1) which represents the ratio of the volatilities in the high and low states is also exhibited in Tables 2 and 3. Note that the estimates of this parameter differ between the normal and the t distribution specifications. In particular, the coefficient is higher in the normal distribution specification and suggests that the volatility in the high state is 9.2 times as large as the volatility in the low state, whereas under the t distribution specification the volatility in the high state is 5.4 times as large as the volatility in the low state. These results can be informally interpreted as providing strong evidence of the existence of regime switches in the conditional variance. The formal hypothesis test procedure will be presented later.

In relation to the persistence of the volatility states, it was found that they exhibit a great degree of persistence as it is suggested by the fact that the transition probabilities P_{11} and P_{22} are close to one in both specifications. In a Markov-switching specification the length of state i is given by the formula $1/(1-P_{ii})$. Since the t specification is preferred, from now on the investigation will centre the discussion on the estimates under this probability distribution. The transition probabilities P_{11} and P_{22} indicate that the low volatility state lasts on average 109 weeks whereas the high volatility state is shorter and has an expected duration of 41 weeks. These results are in line with other studies about Mexican exchange rates that also show a shorter high volatility regime. Also, note that the estimated ergodic probabilities show that the Peso has been in the low volatility state 72.6% of the sample period.

The authors also compare the degree of persistence as measured by the ARCH coefficient α_1 in the SWARCH model, with the corresponding coefficient in a standard AR(1)-ARCH(1)-t model, whose estimates are shown in Table 4. As mentioned before, traditional ARCH models have been criticized because they imply a substantial degree of volatility persistence as measured by the ARCH coefficient. In the estimated AR(1)-ARCH(1)-t model, this coefficient equals 0.633. In contrast, in the SWARCH model the ARCH coefficient has a substantially lower magnitude and equals .2027. These results show that once structural breaks are included the degree of persistence experiences a considerable decrease.

The null hypothesis that there is no regime switching in the ARCH effects of the Peso exchange rate was also formally tested. This is possible because the AR(1)-ARCH(1)-t is nested into the SWARCH model so that a traditional likelihood ratio test (LR) can be used. In other words, the SWARCH model becomes the AR(1)-ARCH(1)-t, if the variance factor g(1) equals 1. Table 5 shows the results of this test, the test statistic is 2(2902.53-2896.66) = 11.74. If the null hypothesis is true, the test statistic has a χ^2 distribution the number of degrees of freedom being equal to the difference in parameters between the two models which is 3. The critical value when the level of significance is .05 is 7.815, so that the null of no switching in the ARCH effects is rejected and it is concluded that the SWARCH model is preferred.

Every week, the realization of the Peso exchange rate is drawn from one of two distinct probability distributions corresponding to two regimes. Graph 2 shows the weekly value of the exchange rate and the probability that a high volatility regime was present on that week. These probabilities are known as the smoothed probabilities and are estimated along with the SWARCH model parameters using the maximum likelihood approach. They are ex-post inferences about the regime in a particular week and are calculated using the complete sample of observations. In total, five episodes of high volatility in the Peso exchange rate were formally identified in the period 1995-2013. A list of these periods and the main financial event associated to them is shown on Table 6. The mean duration of a high volatility period is 48.8 weeks. Note that each of these five periods of high volatility in exchange rates seems to be associated with a financial crisis. In particular, the first of these periods coincide with the Mexican Peso crisis of December 1994 which had a duration of one year (1995). The second episode of high volatility in exchange rates occurred from August 6, 1998 to February 11, 1999 and seems to coincide with the Brazilian crisis which according to official sources extended from November 1998 to March 1999. Interestingly, the Asian crisis which started one year before the Brazilian crisis had no effect on the volatility of the Mexican Peso. However, other papers have documented an effect of this crisis on the Mexican stock market. No other high volatility period of the Peso was identified until the beginning of the global financial crisis of 2008 and 2009. The identified high volatility period ran from August 14, 2008 to January 14, 2010 and it is the longest episode identified by the model with a duration of 75 weeks. After the global financial crisis, two more periods of heightened volatility are identified in the sample period. The first one goes from July 21, 2011 to September 6, 2012 and coincided with a sharp fall in global stock markets due to fears of contagion to Spain and Italy of the European sovereign debt crisis. The studied sample period concludes with a high volatility period that extended from April 11, 2013 to November 7, 2013. Most emerging markets currencies, particularly the Indian Rupee, the Brazilian Real and South African Rand, touched 4-year lows against the dollar in June 2013. The Mexican Peso also experienced a long volatility period over the prospect of imminent tapering of the Federal Reserve's policy of quantitative easing (QE) program.

CONCLUSION

In this investigation, a regime-switching ARCH (SWARCH) for the Peso exchange rate was estimated using both a normal and a t probability distribution. The t distribution specification was preferred since it provided a greater log likelihood. Significant statistical evidence was found supporting the existence of two regimes in the ARCH effects of the Mexican Peso exchange rates. Also, it was found that the volatility in high regimes is 5.4 times as large as the volatility in low regimes and that both regimes are highly persistent as shown by the transition probabilities which are very close to one. However, the SWARCH model which incorporates structural breaks implies a lesser degree of persistence than the traditional ARCH model suggesting that the conjecture of Lamoreaux and Lastrapes that the presence of regime shifts may lead to spurious persistence is valid. Finally, five episodes of heightened volatility in Mexican exchange rates were identified which are mainly associated to international financial crises events. These episodes have an average duration of 41 weeks and are shorter than the low volatility periods.

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TABLES

Table 1: Descriptive statistics of the percentage change in exchange rates in Mexico

Mean	0.09
Standard Deviation	1.83
Skewness	3.8663
Kurtosis	43.6867
Jarque-Bera	80,290**
Ljung-Box (10 lags)	18.137*
Ljung-Box^2 (10 lags)	37.829**
Number of observations	989

Under the null hypothesis of normality, the Jarque-Bera test has a chi-squared distribution with 2 degrees of freedom. The Ljung-Box test for returns with 10 lags has a chi-squared distribution with 10 degrees of freedom. The Ljung-Box test for the squared returns has a chi-squared distribution with 10 degrees of freedom.

** Statistically significant at the 1% level of significance

* Statistically significant at the 10% level of significance

Table 2: Parameter estimates of the SWARCH(2, 1)-N model

μ	-0.0001 (0.0004)
Φ	-0.0157 (0.0231)
α_0	0.00007 (0.0010)
α_1	0.3042 (0.0643)
P_{11}	0.9715 (1.0051)
P_{12}	0.0285 (1.0051)
P_{21}	0.1397 (0.5308)
P_{22}	0.8603 (0.5308)
Ergodic Probability (State 0)	0.8305
Ergodic Probability (State 1)	0.1695
g(1)	9.1851 (1.9194)
LogL	2,873.50

The estimated standard errors are in parentheses. LogL is the value of the log likelihood function; P_{11}, $P_{12,}$ $P_{21,}$ $P_{21,}$ P_{22} are the estimated probabilities of transition; g(1) is the the ratio of the volatility in a "high" state relative to a "low" state.

Table 3: Parameter estimates of the SWARCH(2, 1)-t model

μ	-0.0005 (0.0003)
Φ	-0.0263 (0.0312)
K	4.4236 (0.6289)
α_0	0.00009 (0.00001)
α_1	0.2027 (0.0826)
P_{11}	0.9908 (2.5974)
P_{12}	0.0092 (2.5974)
P_{21}	0.0245 (1.7671)
P_{22}	0.9755 (1.7671)
Ergodic Probability (State 0)	0.7261
Ergodic Probability (State 1)	0.2739
g(1)	5.4475 (1.1430)
logL	2,902.53

The estimated standard errors are in parentheses. LogL is the value of the log likelihood function; P_{11}, P_{12}, P_{21}, P_{21}, P_{22} are the estimated probabilities of transition; g(1) is the the ratio of the volatility in a "high" state relative to a "low" state.

Table 4 Parameter estimates for AR(1)-ARCH(1)-t model

μ	-0.0005 (0.0003)
Φ	-0.0161 (0.0328)
K	3.6030 (0.4381)
α_0	0.0001 (0.0000)
α_1	0.6329 (0.1240)
logL	2,896.655

The estimated standard errors are in parentheses. LogL is the value of the log likelihood function.

Table 5 Test for regime switching in the ARCH effects of the Peso exchange rate

	AR(1)-ARCH(1)-t	**SWARCH(2, 1)-t model**
Log Likelihood	2,896.655	2,902.53
Likelihood Ratio	11.74*	

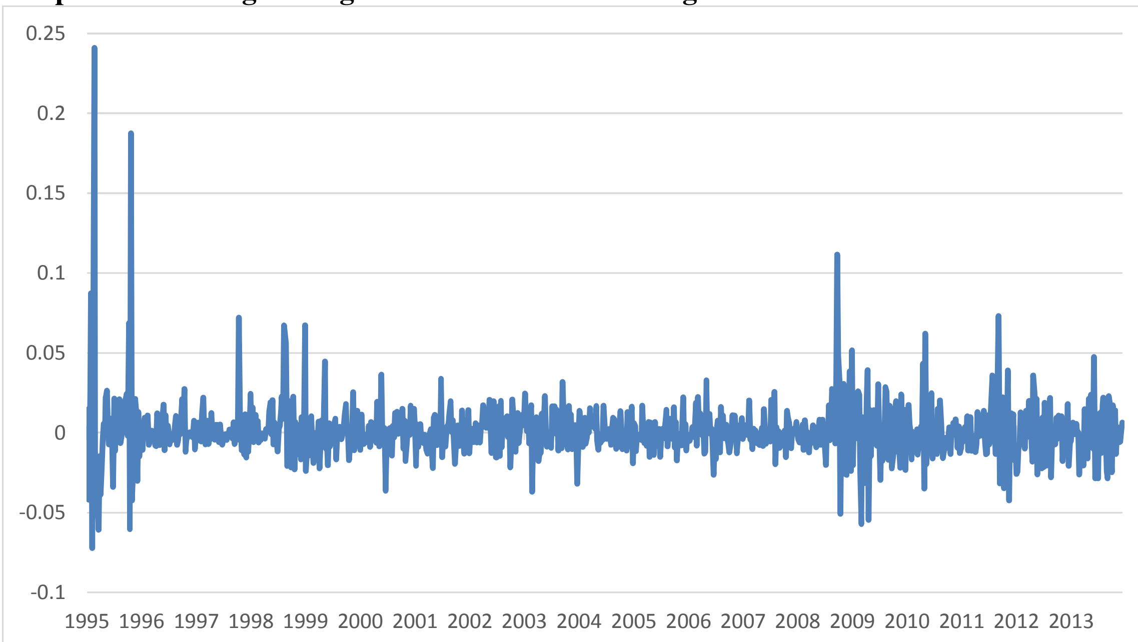
*Statistically significant at the 5% level of significance.

Table 6 Mexican peso high volatility periods

Period	**Duration (weeks)**	**Main event associated to period**
1/19/95-1/4/96	51	Mexican peso crisis
8/6/98-2/11/99	27	Brazilian crisis
8/14/08-1/14/10	75	Global financial crisis
7/21/11-9/6/12	60	European sovereign debt crisis
4/11/13-11/7/13	31	Tapering of Fed's quantitative easing (QE) program

GRAPHS

Graph 1: Percentage change of Mexican Peso exchange rate



Graph 2: Exchange rate and probability of high volatility regime